CORE MATHEMATICS (C) UNIT 2 TEST PAPER 10

1. (i) Given that $y = \log_3 x$, express each of the following in terms of y:

   (a) $\log_3 x^4$, (b) $\log_3 \frac{81}{x}$. [3]

   (ii) Find, to three decimal places, the value of x for which $9^x = 8$. [2]

2. Find all solutions in the interval $0 \le x \le 360$ of the equation

   $$\sin x° \tan x° = 2,$$

   giving your answers to the nearest degree. [6]

3. The numbers 48, x and 3 are the first three terms in a geometric series.

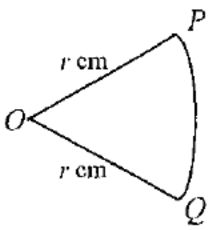
   (i) Find the two possible values of x. [3]

   (ii) For each value of x, find the sum to infinity of the series. [4]

4. In triangle ABC, $AB = 4$ cm, $AC = 3.5$ cm and angle $ABC = 1$ radian. Given that angle ACB is acute, calculate

   (i) the size of angle ACB, in radians to 2 decimal places, [3]

   (ii) the area of triangle ABC, in cm^2 to 1 decimal place. [4]

5. 

   A sector OPQ of a circle of radius r cm has area 100 cm^2.

   (i) Show that the perimeter of the sector is $2r + \frac{200}{r}$ cm. [4]

   (ii) Deduce the value of r for which the perimeter is a minimum [4]

6. A curve C has gradient equal to $2(x + 1)$ at the point (x, y).

   (i) Given that C passes through $(1, 5)$, find the equation of C in the form $y = \mathrm{f}(x)$. [4]

   The straight line $y = x + k$ is the tangent to C at a point P.

   (ii) Find the value of k. [4]

7. (i) Expand $(2 - x)^6$ in ascending powers of x, simplifying each term. [4]

   (ii) Use your answer to part (a) to deduce the expansion of $(2 + x)^6$. [2]

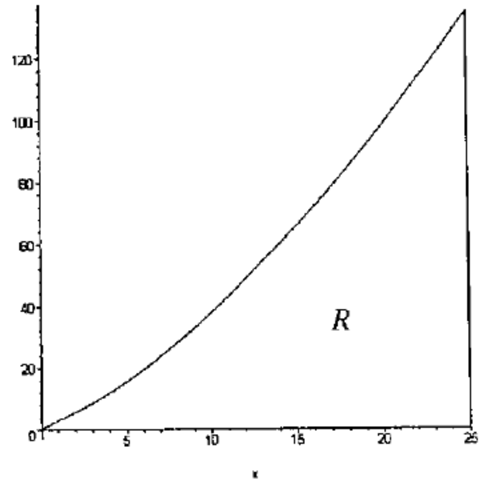
   (iii) Hence, or otherwise, factorise $(2 + x)^6 - (2 - x)^6$ completely. [4]

8. S_1 is the sum of the positive integers from 1 to n inclusive.

   S_2 is the sum of the *odd* positive integers from 1 to $2n + 1$ inclusive.

   Given that $S_2 - S_1 = 66$, find the value of n. [10]

9. The diagram shows the region R bounded by the curve $y = (x+2)\sqrt{x}$, the line $x = 25$ and the x-axis.



   (i) Use the trapezium rule, with five intervals of equal length, to estimate the area of R to the nearest integer. [6]

   (ii) Calculate the true value of this area. [5]

CORE MATHS 2 (C) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1.	(i) (a) $4\log_3 x = 4y$ (b) $\log_3 81 - \log_3 x = 4 - y$	B1 M1 A1	
	(ii) $\log_{10} 8 / \log_{10} 9 = 0.946$	M1 A1	5
2.	$\sin^2 x = 2\cos x$ $\cos^2 x + 2\cos x - 1 = 0$ $(\cos x + 1)^2 = 2$	B1 M1 A1	
	$\cos x = \sqrt{2} - 1 = 0.414$ $x = 66$, $x = 294$	M1 A1 A1	6
3.	(i) $x^2 = 3(48) = 144$ $x = -12$ or $x = 12$	M1 A1 A1	
	(ii) $r = ¼$ or $-¼$ $S_\infty = 48/(5/4) = 38.4$ or $48/(3/4) = 64$	B1 M1 A1 A1	7
4.	(i) $\sin C / 4 = \sin 1 / 3.5$ $\sin C = 0.962$ $\angle ACB = 1.29$	M1 A1 A1	
	(ii) $\angle ACB = 1.293$; $\angle BAC = 0.849$ Area $= 7\sin 0.849 = 5.3$ cm^2	M1 A1 M1 A1	7
5.	(i) $½ r^2\theta = 100$ $\theta = 200/r^2$ Perimeter $= 2r + r\theta = 2r + 200/r$	M1 A1 M1 A1	
	(ii) $dp/dr = 2 - 200/r^2 = 0$ when $r = 10$	M1 A1 M1 A1	8
6.	(i) $y = \int 2x + 2\,dx = x^2 + 2x + c$ $y(1) = 5$ so $c = 2$ $y = x^2 + 2x + 2$	M1 A1 M1 A1	
	(ii) $y = x + k$ has gradient 1, so at P, $2(x + 1) = 1$ $x = -1/2$	M1 A1	
	Then $y = 5/4$, so $k = 7/4$	M1 A1	8
7.	(i) $(2 - x)^6 = 2^6 + 6(2^5)(-x) + 15(2^4)(-x)^2 + 20(2^3)(-x)^3 + 15(2^2)(-x)^4$	M1 A1	
	$+ 6(2)(-x)^5 + (-x)^6 = 64 - 192x + 240x^2 - 160x^3 + 60x^4 - 12x^5 + x^6$	M1 A1	
	(ii) $(2 + x)^6 = 64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$	M1 A1	
	(iii) $(2 + x)^6 - (2 - x)^6 = 384x + 320x^3 + 24x^5 = 8x(3x^4 + 40x^2 + 48)$	M1 A1	
	$= 8x(3x^2 + 4)(x^2 + 12)$	M1 A1	10
8.	$S_1 = ½\, n(n + 1)$ $S_2 = ½\,(n + 1)(2 + 2n) = (n + 1)^2$	M1 A1 M1 A1 A1	
	When $(n + 1)^2 - ½\, n(n + 1) = 66$, $(n + 1)(2n + 2 - n) = 132$	M1 A1 A1	
	$n^2 + 3n - 130 = 0$ $(n - 10)(n + 13) = 0$ $n = 10$	M1 A1	10
9.	(i) (0, 0), (5, 15.652), (10, 37.947), (15, 65.841), (20, 98.387), (25, 135)	B3	
	$½\,(5)(135 + 2(217.827)) = 1427$	M1 A1 A1	
	(ii) $\int_0^{25} x^{3/2} + 2x^{1/2}\,dx = \left[\frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2}\right]_0^{25} = 1250 + 500/3 = 1416\frac{2}{3}$	B2 M1 A1 A1	11